Kinematics: Linear and Circular Motion

PHYS 2425

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1. Position, Velocity, and Acceleration

A. Explain why knowing velocity $v(t)$ determines how position $x(t)$ changes over time, but not its absolute value.

B. Two cars have identical velocity functions $v(t)$ but start from different positions. Sketch and compare their $x(t)$ graphs.

C. Can velocity be zero while acceleration is not? Give an example and sketch the graphs.

D. Can acceleration be zero while velocity is not? Give an example.

E. If $a(t) > 0$ always, what must be true about the velocity graph? Does this guarantee the object moves away from the origin?

F. For $x(t) = 5t$, $x(t) = t^2$, and $x(t) = \sin t$, find $v(t)$ and $a(t)$. Describe how each type of motion feels physically.

G. Suppose $a(t) = 2$, with $v(0) = 3$ and $x(0) = 1$. Find $v(t)$ and $x(t)$.

2. Circular Motion

A. Define uniform circular motion. What is constant and what is changing?

B. A particle moves in a circle of radius r with (instantaneous) speed v. Write expressions for:

1. Angular speed
2. Period and frequency
3. Radial acceleration

C. For uniform circular motion, explain qualitatively why the acceleration vector is nonzero even though the speed is constant.

D. A car of mass m travels around a flat circular track of radius r at constant speed v. Assuming the required centripetal force is provided by static friction, write the inequality that must be satisfied.

$$F_{\mathrm{c}} = m\frac{v^2}{r}, \quad F_{\mathrm{fric,max}} = \mu_s mg$$

$$m\frac{v^2}{r} \leq \mu_s mg \quad \Rightarrow \quad v \leq \sqrt{\mu_s gr}$$

E. Non-uniform circular motion: a particle moves on a circle of radius r with angular position $\theta(t)$. Write the velocity and acceleration vectors.

$$\mathbf{v} = r\dot{\theta}\,\hat{\boldsymbol{\theta}} = r\omega\,\hat{\boldsymbol{\theta}}, \quad \mathbf{a} = r\ddot{\theta}\,\hat{\boldsymbol{\theta}} - r\dot{\theta}^2\,\hat{\mathbf{r}} = r\alpha\,\hat{\boldsymbol{\theta}} - r\omega^2\,\hat{\mathbf{r}}$$

Interpretation: the tangential component $r\alpha$ changes the speed, while the radial component $r\omega^2$ changes the velocity direction.

F. A bead moves on a ring of radius 0.5 m with angular speed $\omega = 4$ rad/s and angular acceleration $\alpha = -2$ rad/s^2. Compute tangential and radial accelerations.

$$a_{\tan} = r\alpha = (0.5)(-2) = -1.0 \text{ m/s}^2$$

(meaning 1.0 m/s^2 opposite the tangential direction)

$$a_{\mathrm{r}} = r\omega^2 = (0.5)(16) = 8.0 \text{ m/s}^2$$

Total acceleration magnitude:

$$|\mathbf{a}| = \sqrt{(-1.0)^2 + (8.0)^2} \approx 8.06 \text{ m/s}^2$$

3. Circular Motion

A. Define uniform circular motion. What is constant and what is changing? Uniform circular motion means the particle moves around a circle at constant speed. The magnitude of velocity is constant, but the direction changes. Therefore, acceleration is nonzero and points toward the center of the circle.
B. A particle moves in a circle of radius r with (instantaneous) speed v. Write expressions for:

1. Angular speed: $\omega = \frac{v}{r}$.
2. Period and frequency: $T = \frac{2\pi}{\omega}, \quad f = \frac{1}{T} = \frac{\omega}{2\pi}$.
3. Radial acceleration: $a_{\mathrm{r}} = \frac{v^2}{r} = r\omega^2$.

C. For uniform circular motion, explain qualitatively why the acceleration vector is nonzero even though the speed is constant.