Vectors
PHYS 2425

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1. Vector Components

Any vector can be written in terms of its components:

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

A. A vector of magnitude 10 makes a 30° angle above the x-axis. Find A_x and A_y.

B. A force of 100 N is applied at 45° above the horizontal. What are the x and y components?

2. Vector Addition

$$\vec{R} = \vec{A} + \vec{B} = (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath} + (A_z + B_z)\hat{k}$$

A. Add $\vec{A} = 3\hat{\imath} + 4\hat{\jmath}$ and $\vec{B} = -2\hat{\imath} + 5\hat{\jmath}$.

B. A displacement of 5 m east and 12 m north is followed by a displacement of 8 m west. Find the resultant displacement vector.

Group Discussion: See if you can find a way to add or subtract vectors *without* splitting them into their components.

3. Dot Product

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z \qquad \vec{A} \cdot \vec{B} = |\vec{A}|\,|\vec{B}| \cos\theta$$

A. Compute $\vec{A} \cdot \vec{B}$ for $\vec{A} = (3, 4)$ and $\vec{B} = (5, 12)$.

B. If $\vec{F} = (10, 0)$ N and $\vec{d} = (2, 2)$ m, what is the work $W = \vec{F} \cdot \vec{d}$?

4. Cross Product

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \qquad |\vec{A} \times \vec{B}| = |\vec{A}|\,|\vec{B}| \sin\theta$$

A. Compute $\vec{A} \times \vec{B}$ for $\vec{A} = (1, 0, 0)$ and $\vec{B} = (0, 1, 0)$.

B. A position vector $\vec{r} = (2, 0, 0)$ m and a force $\vec{F} = (0, 5, 0)$ N are applied. Find the torque $\vec{\tau} = \vec{r} \times \vec{F}$.

Group Discussion: Can you determine the direction of a cross product without performing any calculations?

5. Relevant Tables and Information

Operation	Formula	Result Type
Magnitude	$\lvert\vec{A}\rvert = \sqrt{A_x^2 + A_y^2 + A_z^2}$	Scalar
Addition	$(A_x + B_x,\, A_y + B_y,\, A_z + B_z)$	Vector
Dot Product	$\vec{A} \cdot \vec{B} = \lvert\vec{A}\rvert\lvert\vec{B}\rvert \cos\theta$	Scalar
Cross Product	$\lvert\vec{A} \times \vec{B}\rvert = \lvert\vec{A}\rvert\lvert\vec{B}\rvert \sin\theta$	Vector

Table 1: Vector operations summary.