Vectors Solutions
PHYS 2425

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September 4, 2025

1. Vector Components

Any vector can be written in terms of its components:

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

A. A vector of magnitude 10 makes a 30° angle above the x-axis. Find A_x and A_y.

$$A_x = 10\cos(30^\circ), \quad A_y = 10\sin(30^\circ)$$

B. A force of 100 N is applied at 45° above the horizontal. What are the components?

$$F_x = 100\cos(45^\circ)\text{N}, \quad F_y = 100\sin(45^\circ)\text{N}$$

2. Vector Addition

$$\vec{R} = \vec{A} + \vec{B} = (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath} + (A_z + B_z)\hat{k}$$

A. Add $\vec{A} = 3\hat{\imath} + 4\hat{\jmath}$ and $\vec{B} = -2\hat{\imath} + 5\hat{\jmath}$.

$$\vec{R} = (3-2)\hat{\imath} + (4+5)\hat{\jmath} = \hat{\imath} + 9\hat{\jmath}$$

B. A displacement of 5 m east and 12 m north is followed by a displacement of 8 m west. Find the resultant.

$$R_x = 5 - 8 = -3, \quad R_y = 12$$

$$\vec{R} = -3\hat{\imath} + 12\hat{\jmath}, \quad |\vec{R}| = \sqrt{(-3)^2 + 12^2} = \sqrt{153} \approx 12.37 \text{ m}$$

3. Dot Product

$$\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z \qquad \vec{A} \cdot \vec{B} = |\vec{A}|\,|\vec{B}|\cos\theta$$

A. Compute $\vec{A} \cdot \vec{B}$ for $\vec{A} = (3, 4)$ and $\vec{B} = (5, 12)$.

$$\vec{A} \cdot \vec{B} = (3)(5) + (4)(12) = 15 + 48 = 63$$

B. If $\vec{F} = (10, 0)$ N and $\vec{d} = (2, 2)$ m, what is the work $W = \vec{F} \cdot \vec{d}$?

$$W = (10)(2) + (0)(2) = 20 \text{ J}$$

4. Cross Product

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \qquad |\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin\theta$$

A. Compute $\vec{A} \times \vec{B}$ for $\vec{A} = (1, 0, 0)$ and $\vec{B} = (0, 1, 0)$.

$$\vec{A} \times \vec{B} = (0)(0) - (0)(1)\,\hat{\imath} - \big[(1)(0) - (0)(0)\big]\hat{\jmath} + \big[(1)(1) - (0)(0)\big]\hat{k}$$

$$= 0\hat{\imath} - 0\hat{\jmath} + 1\hat{k} = \hat{k}$$

B. A position vector $\vec{r} = (2, 0, 0)$ m and a force $\vec{F} = (0, 5, 0)$ N are applied. Find the torque $\vec{\tau} = \vec{r} \times \vec{F}$.

$$\vec{\tau} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 0 & 0 \\ 0 & 5 & 0 \end{vmatrix} = (0-0)\hat{\imath} - (0-0)\hat{\jmath} + (10-0)\hat{k} = 10\hat{k}\ \text{N} \cdot \text{m}$$

5. Relevant Tables and Information

Operation	Formula	Example Result
Magnitude	$\lvert\vec{A}\rvert = \sqrt{A_x^2 + A_y^2 + A_z^2}$	$\lvert(3,4)\rvert = 5$
Addition	$(A_x + B_x,\ A_y + B_y,\ A_z + B_z)$	$(3,4) + (5,12) = (8,16)$
Dot Product	$\vec{A} \cdot \vec{B} = \lvert\vec{A}\rvert\lvert\vec{B}\rvert \cos\theta$	$(3,4) \cdot (5,12) = 63$
Cross Product	$\lvert\vec{A} \times \vec{B}\rvert = \lvert\vec{A}\rvert\lvert\vec{B}\rvert \sin\theta$	$(1,0,0) \times (0,1,0) = \hat{k}$

Table 1: Vector operations with worked examples.